Tutorial 5
Operations with Signed Numbers

Introduction

I know that you already can add, subtract, multiply, and divide. However, when negative numbers get involved, things can get a little tricky. There are lots of negatives used in algebra, so this is a good time to review all the things you need to know when you are working with signed numbers. Here we go!

Lesson Objectives

After you go through this tutorial, you should be successful with each of the objectives listed below.
1. Recognize zero pairs.
2. Understand absolute value.
3. Know how to add, subtract, multiply, and divide with signed numbers.

Integers and Absolute Value

Recall from previous discussion that integers are defined as all the whole numbers and their opposites. Another way to say it is that integers are all the positive and negative counting numbers plus zero.

The set of integers in infinite and extends without end to the left and to the right of 0 on the number line. An integer has both a magnitude (how much) and a direction (positive or negative). Typically we do not write the sign (+) when a number is positive.

The Number Line

-6   -4   -2   0   +2   +4   +6
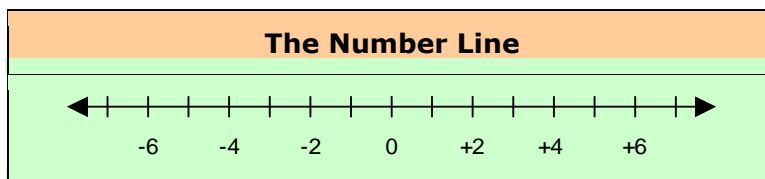
Positive Numbers
Positive numbers are all the numbers greater than zero, that is, to the <u>right of zero</u> on the number line.

Negative Numbers
All numbers less than zero, or to the <u>left of zero</u> on the number line are negative.

Opposites
When two numbers are opposites they are the same distance from zero on the number line, but in opposite directions – one is positive and one is negative. Every whole number, fraction, and decimal has an opposite.

Examples	
3 and -3	2.6 and -2.6
$\dfrac{1}{3}$ and $-\dfrac{1}{3}$	Zero is its own opposite

Zero Pair
The sum of any pair of opposites is always zero, thus we call them a zero pair.

MATH TIP!

The placement of the sign does not affect the value of a negative fraction. In other words, $\dfrac{-1}{2} = \dfrac{1}{-2} = -\dfrac{1}{2}$.

Absolute Value
The absolute value of a number is the distance a number is from zero, <u>regardless</u> of its direction. It is the magnitude of an integer without regard for its direction.

Example
4 and − 4 have the same absolute value. Same distance from zero -6   -4   -2   0   2   4   6
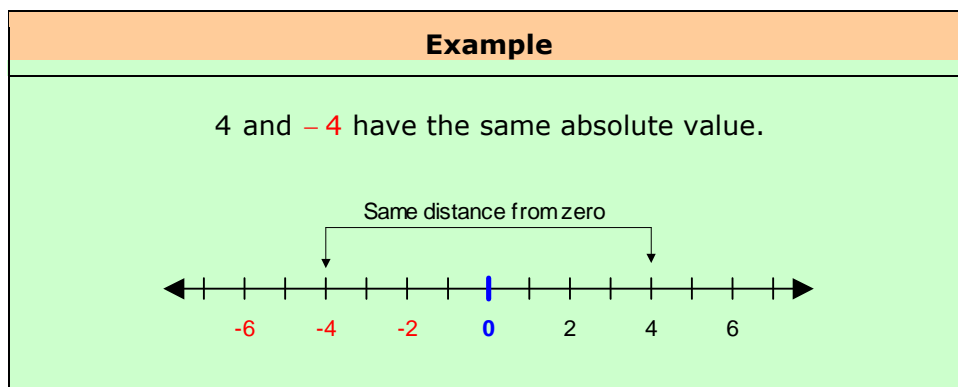
Absolute value is notated by placing the number between two vertical lines (or bars).

Examples	
$\lvert -6 \rvert = 6$	The absolute value of negative six is six.
$\lvert 12 \rvert = 12$	The absolute value of positive twelve is twelve
$\lvert -8 \rvert = \lvert 8 \rvert$	Absolute value is always positive because distance cannot be negative. Therefore opposites will always have the same absolute value.

Adding and Subtracting Signed Numbers

Addition and subtraction are inverse relations – that is, one operation is the opposite of the other. You can say that one will "undo" the other.
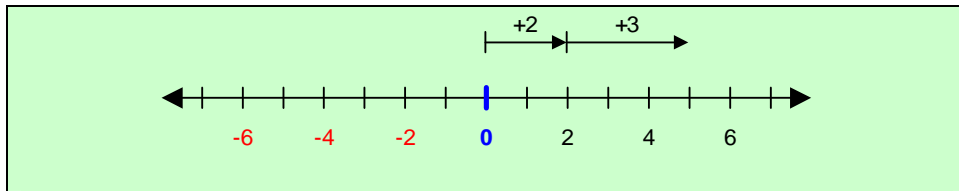
Adding Numbers with the Same Signs
A number gets larger when you **add a positive** number to it. You can see this on a number line. So when you add two positive numbers together, there will be two moves to the right on the number line.

Likewise, a number gets smaller when you **add a negative** number to it. This also can be seen on the number line because negative numbers move to the left. Therefore, if we add two negatives together, the sum will be smaller than either of the individual values as there will be two moves to the left.

We can see from the two examples below that when we add numbers with the same signs, the absolute values of the numbers are being added together and the signs of the numbers are telling us which direction to go on the number line.

Adding Two Positive Numbers
Add: $^+2$ and $^+3$.
Begin at 0 and move 2 units to the right to +2; then move 3 more units to the right, arriving at +5.
$2 + 3 = 5$

+2   +3

```
←―+―+―+―+―+―+―+―|―+―+―+―+―+―+―+→
     -6      -4      -2      0      2      4      6
```

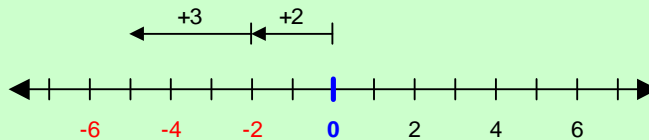## Adding Two Negative Numbers

Add: ⁻2 and ⁻3.

Begin at 0 and move 2 units to the left to ⁻2; then move 3 more units to the left, arriving at ⁻5.

⁻2 + ⁻3 = ⁻5

+3   +2

```
←―+―+―+―+―+―+―+―|―+―+―+―+―+―+―+→
     -6      -4      -2      0      2      4      6
```

**Adding Numbers with Different Signs**
When we add two numbers with different signs, there will be movement in both directions on the number line. You can see from the number line examples that the sum of the two numbers will be the difference between their absolute values, and the result will have the same sign as the sign of the number with the greater absolute value.
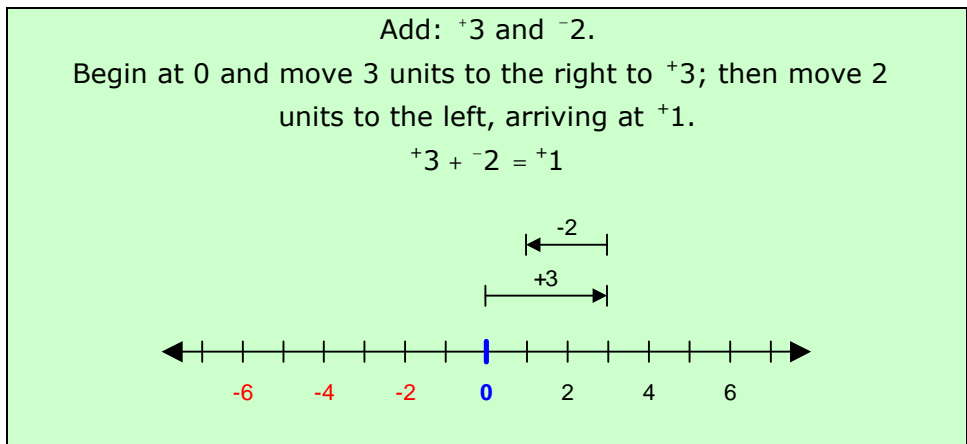
Examples

Add: ⁻3 and ⁺2.

Begin at 0 and move 3 units to the left to ⁻3; then move 2 units to the right, arriving at ⁻1.

⁻3 + ⁺2 = ⁻1

+2

-3

```
←―+―+―+―+―+―+―+―|―+―+―+―+―+―+―+→
     -6      -4      -2      0      2      4      6
```

Add: $^+3$ and $^-2$.

Begin at 0 and move 3 units to the right to $^+3$; then move 2 units to the left, arriving at $^+1$.

$$^+3 + {}^-2 = {}^+1$$
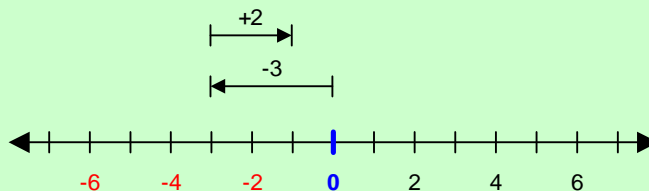
From the examples shown above, we can formulate some simple rules to follow when adding signed numbers.

Rules for Adding Signed Numbers		
Process	**Rules**	**Examples**
Add numbers with like signs	1. Add the absolute values 2. The sum will have the same sign as the addends	$53 + 74 \rightarrow 53 + 74$ $53 + 74 = 127$ $^-53 + {}^-74 \rightarrow 53 + 74$ $^-53 + {}^-74 = {}^-127$
Add numbers with unlike signs	1. Subtract the smaller absolute value from the greater. 2. The sum will have the sign of the number with greater absolute value	$53 + {}^-74 \rightarrow 74 - 53$ $53 + {}^-74 = {}^-21$ $^-53 + 74 \rightarrow 74 - 53$ $^-53 + 74 = 21$

Subtracting Signed Numbers

To subtract signed numbers, we just "add the opposite." Change the subtraction to addition and reverse the sign of the number being subtracted. The just follow the rules for adding signed numbers.

Subtracting Signed Numbers		
$3 - 7 = 3 + {}^-7 = {}^-4$	Positive less a positive.	Subtract a positive, Add a negative
$^-3 - 7 = {}^-3 + {}^-7 = {}^-10$	Negative minus a positive.	Subtract a positive, Add a negative

$3 - {}^-7 = 3 + 7 = 10$	Positive less a negative.	Subtract a negative Add a positive
${}^-3 - {}^-7 = {}^-3 + 7 = 4$	Negative minus a negative.	Subtract a negative Add a positive

Multiplying and Dividing Signed Numbers

Multiplying and dividing with signed numbers is easy. Just remember that when the signs are the same, the result will be positive, and when the signs are different, the result will be negative.

Rules for Multiplying and Dividing Signed Numbers		
Process	**Rule**	**Examples**
Multiply numbers with like signs	Like signed numbers will have a positive product. $(+) \cdot (+) = +$ $(-) \cdot (-) = +$	$3 \cdot 4 = 12$ ${}^-3 \cdot 4 = 12$
Multiply numbers with unlike signs	Unlike signed numbers will have a negative product. $(+) \cdot (-) = -$ $(-) \cdot (+) = -$	$3 \cdot {}^-4 = {}^-12$ ${}^-3 \cdot 4 = {}^-12$
Divide numbers with like signs	Like signed numbers will have a positive quotient. $+\overline{)\,+}$ $-\overline{)\,-}$	$+3\overline{)\,+12}^{\;+4}$ $-3\overline{)\,-12}^{\;+4}$

Divide numbers with unlike signs	Unlike signed numbers will have a negative quotient. $+\overline{)\;-\;}$ $-\overline{)\;+\;}$	$+3\overline{)-12}^{\;-4}$ $-3\overline{)+12}^{\;-4}$

CHECK YOUR UNDERSTANDING